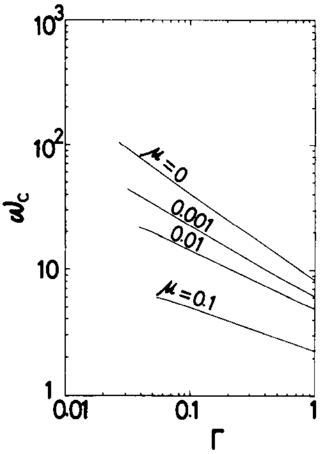

FIG. 3. The critical frequency ω_c as a function of Γ for various values of μ when $P = 100$.

exchange of stabilities does not hold, that is, instability manifests itself as overstability. In order that instability manifests itself as overstability, the value of Γ must be greater than about 0.05 for $\mu = 0.1$. This, recalling that $\Gamma = \lambda_1\kappa/d^2$, means that the thickness d of the liquid layer must be smaller than about 0.5 mm since for most viscoelastic liquids λ_1 is at most 0.1 s [11, 16] and κ is about 0.001 cm s^{-1} [10, 11]. It therefore appears that an experimental investigation under normal laboratory conditions is not feasible. In this regard, however, it should be noted that aqueous solutions of certain recently developed polymers have relatively large relaxation times and rather low viscosities. Perhaps further development of such polymers will make oscillatory convection of more practical concern. It is also seen from Fig. 1 that the critical internal Rayleigh number R_{I_c} for the onset of overstability decreases with increase of Γ and increases with μ. Hence we may say that the elasticity of a viscoelastic liquid has a destabilizing influence on a liquid layer heated internally. It should finally be noted that the results in this note are qualitatively very similar to those in ref. [12] for the classical Bénard problem and in ref. [13] for the electrohydrodynamic instability.

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Evaluation of the importance of the relative velocity during evaporation of drops in sprays

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1. INTRODUCTION

EVAPORATION and combustion of liquid sprays in power systems invariably occurs in environments where there is a convective flow past the spray. This convective flow influences evaporation and combustion in at least two ways. First, it changes the heat and mass transfer rates between the spray as an entity, and the ambience. Secondly, it changes the geometry of the spray by entrainment of the spray periphery and recirculation of the gases surrounding the spray. These processes are all very complex and difficult to model. For this reason, guidance was sought initially from the study of individual drop evaporation and combustion. These studies [1–6] concurred with the experimental observation that a

NOMENCLATURE

$\hat{M}$	average evaporation flux from the spray [g cm^{-2} s^{-1}]
$\hat{m}$	average evaporation rate per drop [g s^{-1}]
n	drop-number density in the spray [cm^{-3}]
$\tilde{R}$	radius of the spray [cm]
R	radius of the droplet [cm]
$\mathcal{S}$	area of the spherical shell through which the evaporation gases go [cm^2]
t_{evap}	evaporation time of each drop in the spray [s]
T_g	temperature of the gases [K]
$\mathcal{V}$	spray volume [cm^3]
V_g	relative velocity between gas and spray [cm s^{-1}]
V_{evap}	mean characteristic velocity of the fuel vapor evaporating from the spray [cm s^{-1}]
Y_{Fv}	mass fraction of fuel vapor.

Greek symbols

ρ_g	gas density in the spray volume [g cm^{-3}]
ρ_l	liquid fuel density [g cm^{-3}]
ϕ	mixture ratio, (mass of air)/(mass of fuel)
ψ	equivalence ratio, ϕ/ϕ_s.

Superscripts

a	ambient, surrounding the spray
$\circ$	initial
sp	in the spray.

Subscripts

a	ambient, at the edge of the sphere of influence, ref. [19]
F	fuel
s	drop surface.

convective flow past an individual drop can affect evaporation and combustion substantially. The next step towards the modeling of convective flows past sprays is to find ways of using the information already available for individual drops [1–6] in relevant ways that will apply to sprays.

For the purpose of modeling evaporation and combustion, a spray cannot be considered merely as a collection of non-interacting individual drops. Numerous experimental [7–10] and theoretical studies [11–19] have shown that droplet interactions are important in regimes of interest in most power systems. Thus, the flow inside a spray is of a very complex nature, and it would be unrealistic to try modeling it for the purpose of finding the flow around each drop. The important issue to be addressed here is to what extent the flow inside the spray is affected by the convective flow past the spray. One way of assessing this effect is to study the extent of penetration of the ambient flow into the spray volume. If the ambient flow does not penetrate into the spray volume, there will not be a resulting relative velocity past the individual drops present in the spray interior. This would mean that the correct way of expressing the effect of convection past that spray is to model the influence of the flow on the periphery of the spray and study the change of the shape of this periphery due to entrainment.

In the case of a burning spray with an envelope flame, the ambient flow certainly does not penetrate the spray volume to any appreciable extent. Rather, the gaseous fuel evaporated from the drops is transported outside the spray volume to meet the oxidizer and burning takes place in the flame. Vigorous flames such as those encountered in power systems will consume almost all fuel and oxidizer reaching them, thus leaving no appreciable chance for oxidizer to leak through them.

2. MATHEMATICAL MODEL

To estimate the importance of relative velocity past individual drops composing a spray, we use the configuration shown in Fig. 1. The spray is assumed spherical and composed of monodisperse, uniformly distributed droplets. Since the

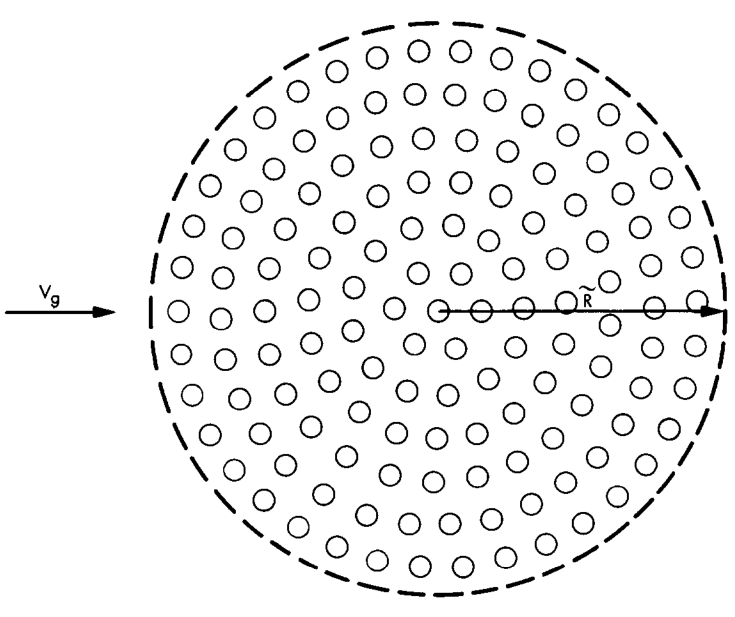

FIG. 1. Spherical monodisperse spray configuration. The droplets are uniformly distributed in the spray volume.

shape of the spray is assumed to stay the same, the underlying hypothesis is that all drops move at the same speed. Thus, the more realistic situation of a spray deforming its shape when injected into a combustion device is not treated here. Rather, this model would better apply to pockets of drops that are moved around by turbulent eddies without having experienced much deformation. Our coordinate system is fixed with the spray, so that V_g represents the relative velocity between the ambient flow and the spray; V_g is assumed constant.

The question that is addressed here is the following: what is a condition for the ambient gas flow to go around the spray without penetrating inside it? Assuming that the pressure inside the spray is equal to that in the ambient flow [19], a sufficient condition for this situation to occur is:

$$\rho_g^{sp} V_{evap}^2 > \rho_g^a V_g^2 \tag{1}$$

which is to say that the relative momentum of the approaching gases should be smaller than the mass efflux from the droplet cloud. But

$$V_{evap} = \hat{M}/\rho_g \tag{2}$$

and

$$\hat{M} = \hat{m} n / \mathcal{V} / \mathcal{S}. \tag{3}$$

Also

$$\hat{m} = \frac{4\pi R^{\circ 3}}{3} \rho_l \frac{1}{t_{evap}}. \tag{4}$$

Thus, since

$$\mathcal{V} = \frac{4\pi \tilde{R}^3}{3} \tag{5}$$

and

$$\mathcal{S} = 4\pi \tilde{R}^2 \tag{6}$$

one obtains

$$V_{evap} = \frac{4\pi}{9} \frac{\rho_l}{\rho_g} R^{\circ 3} n \frac{\tilde{R}}{t_{evap}}. \tag{7}$$

This means that equation (1) can be expressed as

$$t_{evap} < \frac{4\pi}{9} \frac{\rho_l}{\rho_g} R^{\circ 3} n \frac{\tilde{R}}{V_g} \sqrt{\rho^{sp}/\rho^a}. \tag{8}$$

Equation (8) is a sufficient condition for the ambient flow to go around the spray without penetrating it. If this condition is satisfied, the drops inside the spray will not be exposed to a gas flow resulting from the convective flow past the spray; instead, the flow of ambient gases will go around the entire spray and only the drops at the periphery of the spray will have a portion of their surface exposed to that flow.

Estimates of equation (8) can be made using any theory of spray evaporation that appropriately describes this phenomenon for a wide range of parameters such as R°, ϕ, T_{gs}°, T_{ga}°, Y_{Fva}°, and a range of realistic values for V_g and $\tilde{R}$. The spray evaporation model chosen here is that of Bellan and Cuffel [19]. This model assumes that: the gas-phase is quasi-steady with respect to the liquid phase; the drop temperature is a function of time only; the temperature is a continuous function at the droplet surface; thermodynamic equilibrium exists at the drop's surface; all dependent variables are averaged in the space between the spheres of influence (as defined in ref. [19]); the Lewis number of the gas phase is unity; the product of the gas phase density and diffusivity is a constant; the heat capacities at constant pressure and the thermal conductivities in the liquid and gas phase are both constant; ρ_g is time dependent but uniform; ρ_l is constant; the spray is not exposed to body forces; the Mach number of the gas phase is much smaller than unity; radiative and other heat-loss mechanisms are neglected. Calculations performed with this model show that for evaporation the value of $\sqrt{\rho^{sp}/\rho^a}$ is very close to unity, except for very dense sprays where it deviates (increases) by some percentage from unity. Since the present estimate is claimed to be qualitatively, rather than quantitatively, correct, the value of $\sqrt{\rho^{sp}/\rho^a}$ will be taken as unity. This choice provides a conservative estimate of the RHS of equation (8), in the sense that it restricts it.

The LHS of equation (8), t_{evap}, is taken from the results obtained in ref. [19]. Since the evaporation time given by a model which would account for convective flow past each drop of the spray would be smaller than the evaporation time obtained with a model that does not account for convection, by using the model of ref. [19] which ignores convective effects, a conservative value of t_{evap} is obtained for use in equation (8).

3. RESULTS AND DISCUSSION

To estimate the RHS of equation (8), V_g is taken as 5×10^3–15×10^3 cm s^{-1} and $\tilde{R}$ as 50–100 cm; these values are representative of boiler and furnace operation. The calculations were performed for a few ratios $\tilde{R}/V_g$ that could be considered characteristic in these situations.

Consistent with the fact that in the evaporation model $nR^{\circ 3}$ is invariant for fixed ψ, T_{gs}°, T_{ga}° and Y_{Fva}°, as pointed out in ref. [19], R° is here fixed to be 2×10^{-3} cm. Notice that since the RHS of equation (8) is proportional to $nR^{\circ 3}$, the results will be invariant should R° change. The ratio ρ_l/ρ_g is taken to be 5×10^2, a conservative value. All other values of the transport properties are given in ref. [19].

The results are presented in Figs. 2–5. Since equation (8) is a sufficient condition, in reality wider regions than those shown in Figs. 2–5 might not be vulnerable to flow penetration.

Figure 2 shows plots of both t_{evap} and the RHS of equation (8) vs the equivalence ratio, ψ. The regions of no flow penetration in the spray volume are $\psi < 1.56$ for $\tilde{R}/V_g =$

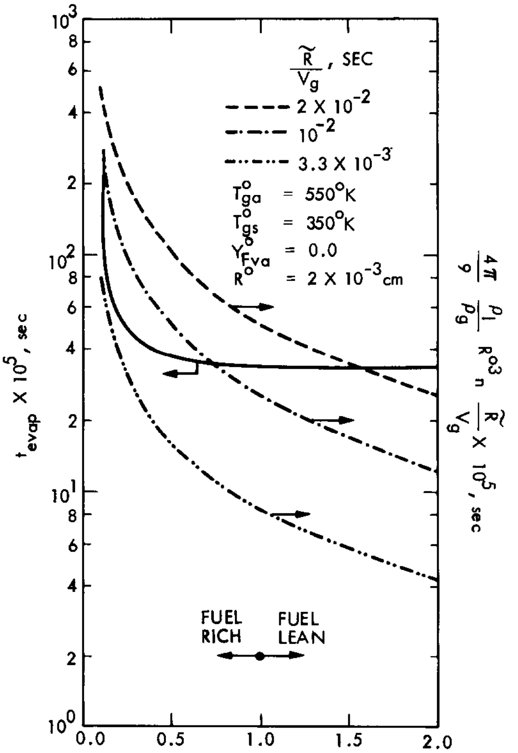

FIG. 2. t_{evap} and RHS of equation (8) vs the equivalence ratio for various values of $\tilde{R}/V_g$. n varies from 4.21×10^4 to 2.26×10^3 cm^{-3}.

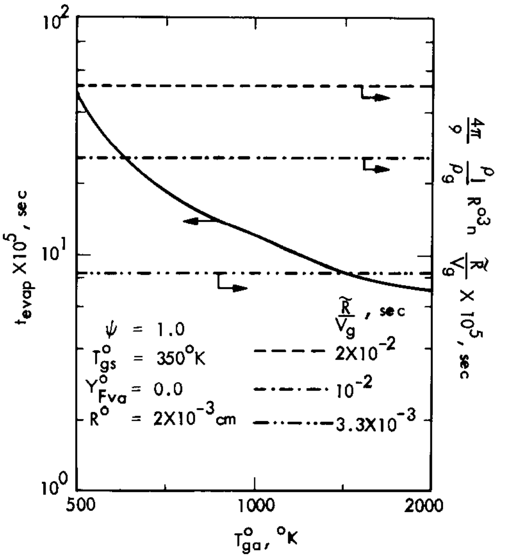

FIG. 3. t_{evap} and RHS of equation (8) vs T°_{ga} for various values of $\tilde{R}/V_g$. n varies from 4.54×10^3 to 4.56×10^3 cm^{-3}.

2×10^{-2} s and $0.12 < \psi < 0.76$ for $\tilde{R}/V_g = 10^{-2}$ s. No such region exists for $\tilde{R}/V_g = 3.3 \times 10^{-3}$ s. The general requirement for no ambient flow penetration is that evaporation be vigorous in order to counter the incoming flow. This is why the criterion set by equation (8) becomes satisfied for richer mixtures and denser sprays. However, if the spray is so dense that evaporation becomes hindered by fuel vapor build-up in the gas phase, the criterion is not satisfied any longer. This happens for $\psi < 0.12$ for $\tilde{R}/V_g = 10^{-2}$ s. In the limit of leaner mixtures, in particular for $n = 1$ cm^{-3}, the effect of ambient flow will be felt past each drop.

Another limit to be checked is that of a cloud of particles which does not evaporate. Then $t_{evap} \rightarrow \infty$ and thus equation (8) is never satisfied which means that flow will be felt past each drop. This result is expected intuitively.

In agreement with the results derived from Fig. 2, Fig. 3 shows that equation (8) is satisfied for larger evaporation rates; these are obtained as T°_{ga} increases. Thus, the criterion is satisfied for $T^{\circ}_{ga} > 1500$ K when $\tilde{R}/V_g = 3.3 \times 10^{-3}$ s and for $T^{\circ}_{ga} > 630$ K when $\tilde{R}/V_g = 10^{-2}$ s; it is always satisfied for $\tilde{R}/V_g = 2 \times 10^{-2}$ s. Since realistic temperatures in boilers and furnaces are 1000–2000 K [20], even conservative estimates show that many regions of the spray will not be penetrated by ambient flow.

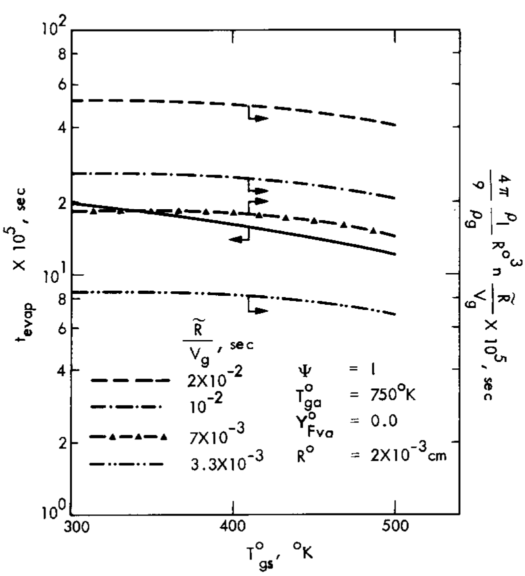

FIG. 4. t_{evap} and RHS of equation (8) vs T°_{gs} for various values of $\tilde{R}/V_g$. n varies from 4.57×10^3 to 3.62×10^3 cm^{-3}.

Figure 4 shows that equation (8) is always satisfied for the two larger values of $\tilde{R}/V_g$ and never satisfied for the smallest value of $\tilde{R}/V_g$. Additional calculations were made for an intermediary value of $\tilde{R}/V_g$ to show that flow penetration is facilitated by lower values of T°_{gs} which hinder evaporation.

The effects of initial vitiation in the gas phase are shown in Fig. 5. Since ψ is fixed throughout the variable Y°_{Fva} calculations, a larger Y°_{Fva} implies a smaller n. For small values of Y°_{Fva}, n is large, evaporation is vigorous and thus equation (8) has a better chance of being satisfied, as clearly seen in Fig. 5. For large Y°_{Fva} and small n, although the evaporation time only slightly increases, the criterion is not satisfied and flow easily penetrates the spray volume. For $Y^{\circ}_{Fva} = 5.9 \times 10^{-2}$, $n = 1.65$ cm^{-3} and despite evaporation still taking place, the spray volume is penetrated by the ambient flow and resulting, convective effects past individual drops should be taken into

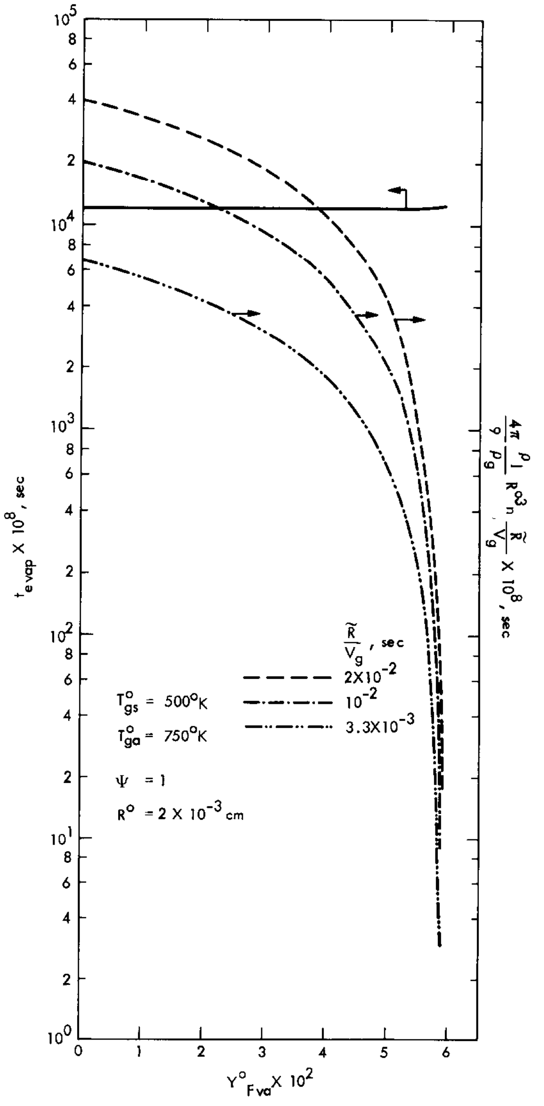

FIG. 5. t_{evap} and RHS of equation (8) vs Y°_{Fva} for various values of $\tilde{R}/V_g$. n varies from 3.62×10^3 to 1.65 cm^{-3}.

account. Thus, dilute sprays are prone to ambient flow penetration, as expected.

In conclusion, the criterion presented herein is a sufficient condition for ambient flow to go around a given spray configuration, rather than penetrating it. Numerical calculations show that a spray will be more prone to ambient flow penetration for leaner mixtures, lower initial temperatures of gas phase and droplet, and more dilute spray configurations.

Similar calculations to those presented above based now upon a dynamic, rather than a static, criterion would greatly contribute to physical accuracy when imbedded in more complicated spray-in-turbulent-flow models. This is because in turbulent flows, eddies contain pockets of droplets that tend to have their own identity much as the cloud in this model. In order to properly model the evaporation of these pockets of drops, it is first necessary to determine the conditions to which they are exposed, in particular if they are exposed to a flow past them. The model developed above shows that such an estimate can be easily accomplished.

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On the relativistic temperature transformations and the related energy transport problem

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A long-standing difficulty extensively discussed in the literature pertains to the correctness of the transformation of temperature in special relativity [1, 2]. With the symbol T^r for the rest frame temperature (rest with respect to the center of mass of the system), the transformation is

$$T' = T^r\sqrt{1-v^2/c^2} \qquad (1)$$

which is occasionally called the Einstein–Planck transformation although the alternative formula, equation (2), was also used by Planck [1]. Here v is the convection velocity of the system. Ott [3] and Arzelies [4] criticized the above expression and advocated the covariant transformation which is characterized by four-vector representations of heat and

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